The **Wallace-Bolyai-Gerwein** theorem states that any two polygons in the plane with equal area are “scissors congruent;” i.e., you can cut one polygon into pieces which can be perfectly fit together (no holes, no overlaps) to form the other. I am indebted to Lalit Jain, a high school teacher at the time, who taught me about this at a workshop in Berkeley.

1. Prove the formula for the area of a triangle in as many ways as possible, including using paper and scissors. Does your proof work for any triangle? Why does it work?

2. Why does the area of a parallelogram only depend on its height and base? Explain this in more than one way.

3. Can any polygon be dissected into triangles? Why? Have you examined both the convex and non-convex cases?

4. Show that two rectangles of equal area are scissors congruent.

5. Do the above problems allow you to prove the WBG theorem?

6. The WBG theorem deals with 2-dimensional polygons. What about other 2-dimensional objects? And what about 3-dimensional shapes?

**Other Area and Dissection Problems**

7. Do you know any proofs of the Pythagorean Theorem that use dissections? What does this have to do with the WBG theorem?

8. **Medians.** We all know that the three medians of a triangle (the lines going from a vertex to the midpoint of the opposite side) intersect in a point and that the intersection point cuts the medians in a 1 : 2 ratio. Accept, for the moment, that the medians meet in a point (although that is worth proving from scratch), but use your knowledge about area to deduce the 1 : 2 ratio, with ease!
9 *Threedians.* The lines $AD, BE, CF$ below are "threedians;" in other words, they hit the opposite edges at a trisection point ($CE = AC/3$, etc.). What can you say about the relationship of the shaded area $[GHI]$ to the area $[ABC]$?

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10 *Infinite dissections.* (Thanks to Sam Vandervelde.) Can you dissect a square into infinitely many line segments? Of course you can. (A line segment, by the way, is straight and has two endpoints and infinitely many points in-between; in other words, it has positive, non-zero length. A single point is *not* a line segment. And “dissecting into line segments” means decomposing into disjoint line segments (every point of the target shape is covered by a line segment, and no two line segments have any point in common, and no other points are covered). So using this definition of line segment, here are a few harder questions. Which of the following can you dissect into infinitely many line segments?

(a) A rectangle.
(b) A trapezoid.
(c) A triangle.
(d) A semicircle.
(e) A circle.